



University of Adelaide
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Discrete Morse Theory and L^2 Homology

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Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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Abstract

A brief overview of Forman's discrete Morse theory is presented, from which analogues of the main results of classical Morse theory can be derived for combinatorial Morse functions, these being functions mapping the set of cells of a CW complex to the real numbers satisfying some combinatorial relations. The discrete analogue of the strong Morse inequality was proved by Forman for finite CW complexes using a Witten deformation technique.

This deformation argument is adapted to provide strong Morse inequalities for infinite CW complexes which have a finite cellular domain under the free cellular action of a discrete group. The inequalities derived are analogous of the L^2 Morse inequalities of Novikov and Shubin ([21]), and the asymptotic L^2 Morse inequalities of an inexact Morse 1-form as derived by Mathai and Shubin in [18].

Contents

1	Introduction	3
2	Preliminaries	6
2.1	CW complexes	6
2.2	The associated chain complex	7
2.3	Regular faces	8
3	Discrete Morse theory	9
3.1	The discrete Morse function	9
3.2	Examples of discrete Morse functions	10
3.3	Equivalence of combinatorial Morse functions	10
3.4	Combinatorial vector fields	12
3.5	Analogues of Morse theory results	15
4	L^2 discrete Morse theory	17
4.1	The L^2 chain complex	17
4.2	Construction of the combinatorial Witten complex	20
4.3	The combinatorial L^2 Morse inequality	24
5	Asymptotic Morse inequalities	30
5.1	Non- Γ -invariant combinatorial Morse functions	30
5.2	The deformed complex	32
5.3	The asymptotic Morse inequality	34
6	Future directions	39
6.1	Asymptotic inequalities	39
6.2	Non-gradient vector fields and Lyapunov functions	42
6.3	The extended category of Farber	44

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